ESTIMATING TERM STRUCTURE OF INTEREST RATE: A PRINCIPAL COMPONENT, POLYNOMIAL APPROACH

by

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CERTIFICATE

This Thesis, presented by **Nasir Ali Khan** under the direction of their supervisor and approved by thesis examination committee, has been presented to and accepted by Department of Statistics, in partial fulfillment of the requirement for the degree of **B.S. in Actuarial Sciences & Risk Management.**

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ABSTRACT

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Polynomial functions of the term to maturity have long been used to provide a general function form for the zero coupon yield curve. I propose a new zero coupon yield curve functional form consisting not of simple polynomial of term, *t*, but rather constructed from 1/(1+t). I model zero-coupon yield as a linear function of the *k* principal components of *n* polynomials of 1/(1+t). The principal component of polynomials of 1/(1+t) model is applied to PIB bonds data.

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1.0 INTRODUCTION

1.1 Term Structure of Interest Rate

The term of interest rate at any time is the relationship of bond maturity and bond yield of similar instruments. It is constructed by graphing time to maturity against yield to maturity for the bond with the same credit risk. The term structure of interest rates, also known as the yield curve, is a very common bond valuation method. Constructed by graphing the yield to maturities and the respective maturity dates of benchmark fixed-income securities, the yield curve is a measure of the market's expectations of future interest rates given the current market conditions. Treasuries, issued by the federal government, are considered risk-free, and as such, their yields are often used as the benchmarks for fixed-income securities with the same maturities. The term structure of interest rates is graphed as though each coupon payment of a fixed-income security were a zero-coupon bond that "matures" on the coupon payment date. The exact shape of the curve can be different at any point in time.

1.2 Pricing Bonds

The price of a bond is the present value of all its cash flows. Therefore, the price a bond is equals to the sum of the present value of all the coupon interest payments and the present value of the face value. The price of a bond that pays yearly coupon can therefore be given by:

$$P = \frac{C}{1+r} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \ldots + \frac{C+M}{(1+r)^N}$$

1.0-1

P = Price of a bond

C = Coupon payment

r = Discount rate or Required yield

N = Numbers of year to maturity

M = Face value of bond

The simplest measure of the yield on a bond is the current yield. Current yield is ratio of coupon and the current price of a bond. It essentially calculates the bond coupon income as a proportion of the price.

Yield to Maturity (YTM) is the periodic interest rate that equates the present value of the future cash flows to be received on the bond to the initial investment on the bond or its current price. This means that yield to maturity is the internal rate of return (IRR) on the bond investment.

To calculate the yield to maturity, the coupon, numbers of years to maturity and the face value of the bond are known, and the discount rate or yield to maturity is the variable to be determined.

$$P = \frac{C_1}{1+YTM} + \frac{C_2}{(1+YTM)^2} + \frac{C_3}{(1+YTM)^3} + \ldots + \frac{C_N + M}{(1+YTM)^N}$$

$$P = \sum_{t=1}^{N} \frac{C}{(1+YTM)^t} + \frac{M}{(1+YTM)^N}$$

1.0-2

The yield for a zero-coupon bond or zero-coupon rate is also called the spot rate for the maturity of the bond. For the zero-coupon bonds, the yield to maturity and the spot rate are equal. When considering coupon-paying bonds, the yield to maturity is a weighted average of the spot rates corresponding to each coupon payment. When valuing bonds, the spot rate is commonly used to calculate the present value of the cash flows because there is no concern about the reinvestment rate for the coupons received over time.

1.3 Data

Data is based on the market yields on 12 March 2004 for a set of traded Pakistan Investment Bonds (PIB). The data consisted of yield and specification for 28 coupon-paying bonds. The term of the bonds ranged from about 6 months to 15 years. The coupon rate ranged from 1.48% to 8.64%

1.4 Methodology

This paper proposes a new and relatively simple method for deriving a zero-coupon bond yield Curve from the market prices of coupon paying treasury bonds.

A coupon bond may be priced a number of ways. The traditional procedure is to discount all of the bonds cash flow at the market-determined yield to maturity.

$$P(y^m) = \frac{C_1}{(1+y^m)} + \frac{C_2}{(1+y^m)^2} + \ldots + \frac{C_t + M}{(1+y^m)^t}$$

1.0-3

Where $P(y^m)$ is the price of a period bond when the yield to maturity is y^m, C_t is the coupon payment at time t and M is the bond's face value.

A coupon bond is a bundle of zero coupon bonds with each coupon payment constituting a single zero bonds. An alternative pricing method uses constituent zero coupon rates. A bond's cash flows are discounted with the relevant zero coupon rate, rather then yield to maturity, provided by the prevailing zero coupon yield curve. Here,

$$P(y^z) = \frac{C_1}{1+y^z} + \frac{C_2}{(1+y^z)^2} + \ldots + \frac{C_t + M}{(1+y^z)^t}$$

1.0-4

Where y^z is the zero coupon rate applicable t a term of *ith* periods.

The zero coupon yield curve y^z, may be estimated as a particular function of the term, t, that minimizes the sum of the squared differences between the actual market bond prices and zero coupon bond price. The zero coupon yield is estimated by minimizing,

$$SSE = \sum_{i=1}^{n} \left(P(y^m) - P(y^z(t)) \right)^2$$

1.0-5

Zero coupon rates are rarely directly observable in financial markets. Attempting to extract zero-coupon rates from the prices of those risk-free coupon-bearing instruments, which are observable, namely government bonds, various models and numerical techniques have been developed. Such models can broadly be categorized into ***parametric*** *and* ***spline-based approaches****,* where a different trade-off between the flexibility to represent shapes generally associated with the yield curve (*goodness-of-fit*) and the *smoothness* characterizes the different approaches

Parametric model, the method developed by Nelson and Siegel (1987) attempts to estimate these relationships by fitting for a point in time *t* a discount function to bond price data by assuming explicitly the zero-coupon or spot interest rate curve can be derived by integrating the forward rate curve:

$$y_z(t) = \beta_0 + (\beta_1 + \beta_2)\frac{t}{m}\left(1 - \exp(-\frac{m}{t})\right) - \beta_2 \exp\left(\frac{-m}{t}\right)$$

1.0-6

In this equation m denotes time to maturity, t the time index and $\beta_{t,0}$, $\beta_{t,1}$,$\beta_{t,2}$ and $\tau_{t,1}$ are parameters to be estimated. For long maturities, spot and forward rates approach asymptotically the value b_0, which must be positive. ($\beta_0 + \beta_1$) determines the starting value of the curve at maturity zero; b1 thus represents the deviation from the asymptote β_0. In addition, ($\beta_0 + \beta_1$) must also be positive. The remaining two parameters β_2 and τ_1 are responsible for the "hump". The hump's magnitude is given by the absolute size of β_2 while the sign give its direction: a negative sign indicates a U-shape and a positive sign a hump. τ_1, which again must be positive, determines the position of the hump.

To improve the flexibility of the curves and the fit, Svensson (1994) extended Nelson and Siegel's function by adding a further term that allows for a second "hump". The extra precision is achieved at the cost of adding two more parameters, β_3 and τ_2, which have to be estimated.

Spline-based methods fit the yield curve by relying on a piecewise polynomial, the spline function3, where the individual segments are joined smoothly at the so-called knot points.

Over a closed interval, a given continuous function can be approximated by selecting an arbitrary polynomial, where the goodness-of-fit increases with the order of the polynomial. Higher-order polynomials, however, quite frequently display insufficient smoothing properties. This problem can be avoided by relying on a piecewise polynomial whereby the higher-order polynomial is approximated by a sequence of lower-order polynomials. Consequently, spline functions are generally based on lower-order polynomials (mostly quadratic or cubic). A cubic spline, for instance, is a piecewise cubic polynomial restricted at the knot points such that their levels and first two derivatives are identical. One parameter corresponds to each knot in the spline. The McCulloch (1975) method uses cubic spline to approximate the discount function. The spline is estimated using ordinary least squares. The mean price is used as the dependent variable and the fitted price errors are weighted by the inverse of the spread. The numbers of parameters are moderate; however the functional form lead to discount rates that tend to positive or negative infinity when extrapolated. In the case of "smoothing splines", the number of parameters to be estimated is not fixed in advance. Instead, one starts from a model, which is initially over-parameterized. Allowing for a large number of knot points guarantees sufficient flexibility for curvature throughout the spline. The optimal number of knot points is then determined by minimizing the ratio of a goodness-of-fit measure to the number of parameters. This approach penalizes for the presence of parameters, which do not contribute significantly to the fit. It is not convenient to draw on the (varying number of) parameters in disseminating yield curve information.

Nelson & Siegal specified the yield curve as a four parameter Laguerre function:

$$y^z(t) = \beta_0 + (\beta_1 + \beta_2 t)e^{\beta_3 t}$$

1.0-7

The Laguerre yield curve has many beneficial aspects. Its flexibility allows for a number of yield curve shapes such as trenches and humps. It also produces sensible rates at the extremities of the term structure, y (0) = $\beta_0+\beta_1$ and y (∞) = β_0. For many yield curve shapes, the non-linear least square estimation process necessary to estimate (4) fails to converge Laguerre form. An alternative to the Laguerre form is the polynomial yield curve. For example, a four-parameter polynomial yield curve is specified as:

$$y_z(t) = \beta_0 + \beta_1 t + \beta_2 t^2 + \beta_3 t^3$$

1.0-8

Polynomial zero coupon yield curve models invariably produces convergence in the estimation process. This form is capable of providing all commonly observed yield curve shapes. The polynomial form is general function that provides a good approximation to any yield curve function, as the tailor series expansion shows. The primary failing of the polynomial form concerns the rate at the long end of the term structure.

I suggest a polynomial of a simple transformation of the term, t, that removes the problem of the long rate instability inherent in this simple polynomial model (5) while retaining the tractable estimation property of the polynomial from.

Polynomial of 1/(1+t) are well behaved in that they approach a constant as t increases. We propose the following zero coupon yield curve;

$$y^z(t) = \beta_0 + \beta_1 \frac{1}{(1+t)^1} + \beta_2 \frac{1}{(1+t)^2} + \ldots + \beta_{p-1} \frac{1}{(1+t)^{p-1}}$$

1.0-9

This paper model zero coupon yield as a function of the principal component of polynomial of 1/ (1+t), where t is the term of zero coupon instrument. We suggest a new principal component approach to selecting a k parameter model from a possible p polynomial terms. This approach employs principal components of polynomials of 1/(1+t). Specifically, our approach uses the best k principal components extracted from p polynomials of 1/(1+t)

This functional form not only has the flexibility, generality and tractable estimation properties of all polynomial forms, but also, and most importantly, the zero-coupon yields, implied by the models, approach some long term as the term increases.

2.0 PRINCIPAL COMPONENT ANALYSIS

2.1 Introduction

Many variables are characterized by a high degree of co linearity between the returns. Collinearity is when the returns process exhibits a very high level of correlation. Collinear systems occur when there are only a few important sources of information in the data, which are common to many variables. Principal Components Analysis (PCA) is a method for extracting the most important uncorrelated sources of information in the data. The objective of PCA is to reduce the dimensionality of the problem so that only the most important sources of information are used, that is, by taking only the first m principal components. Principal components are a set of variables that define a projection that encapsulates the maximum amount of variation in a dataset and is orthogonal 1 (and therefore uncorrelated) to the previous principal component of the same dataset. PCA is very useful in highly correlated systems, since the variation in the system can be explained by just a few principal components, which are the independent sources of variation in the system. The lack of correlation between the principal components results in dimension reduction of the system by taking only a few of the principal components.

2.2 Theoretical Framework

The goal is to find the eigenvectors of the covariance matrix. These eigenvectors correspond to the directions of the principal components of the original data; their statistical significance is given by their corresponding eigenvalues. The principal components are linear combinations of the original data series, that is, they are obtained by multiplying X (the matrix of the original data values) by the matrix of eigenvectors. The original series X is said to be regressed onto a small number of principal components. In this way, PCA implements a dimensionality reduction because it allows one to retain only a small number of components. By choosing the components with the largest variance, a large portion of the total variance of X can be explained. At this point, a reduced number of principal components yield a simplified regression, which results in a more robust estimation of the covariance matrix of the original data series though only a fraction of the variance is explained.

A combination of the variables must be found that explains as much as possible f the original data. This combination is called the first principal component, and n be thought of as a single axis in space. When each observation is projected on that axis the resulting values form a new variable with highest variance among all possible choices of the first principal component. A combination is then found which explains as much as possible of what remains behind, that being the second principal component - and it is another axis in space, which is perpendicular to the first principal component. Projecting the observations onto this axis generates another new variable with highest variance among all possible choices of the second principal component. Proceeding in this way all m principal components can be found.

2.3 PCA Method

The data that is input to PCA must be stationary. Prices, rates or yields are generally non-stationary, and will then have to be transformed, commonly into returns, before PCA is applied. The input into PCA is a correlated system of k stationary time series (order of polynomial), that is, a $T \times k$ stationary data matrix X, where T represents the time points (term to maturity). PCA is based on an eigenvalue and eigenvector analysis of the $k \times k$ symmetric matrix of covariance between the variables in X. The symmetric covariance matrix is given by

$$V = X'X \qquad \textbf{2.0-1}$$

Consider a data of T rows (terms) of n column (polynomials). Let X denotes the $T \times n$ data matrix. The aim is to find a linear combination of the observed asset returns that explains as much as possible of the observed variability in the data. It will be shown that the weights in the linear combination can be chosen from the set of eigenvectors of the covariance matrix. Denote by W the $n \times n$ matrix of eigenvectors of V. Thus Λ is the $n \times n$ diagonal matrix of eigenvalues of V. The columns of W are then ordered according to the size of corresponding eigenvalue. That is, the eigenvalues are sorted in descending order and then the columns of the eigenvector matrix are ordered corresponding to the eigenvalues. Thus if W = (wij) for $i, j = 1,\ldots,k$, then the mth column of W, denoted $w_m = (w_{1m}, \ldots, w_{Tm})$, is the $T \times 1$ eigenvector corresponding to the eigenvalue λ_m and the column labeling has been chosen such that $\lambda_1 > \lambda_2 > \cdots > \lambda_k$. The first column of W maximizes the explained variance; similarly, the

second column of W maximizes the explained variability in the data, given the explanation already provided by the first column. Since the eigenvectors are orthogonal to each other, the principal components obtained as the linear combinations using the eigenvectors as the weights, will be uncorrelated with each other. Much of the total system variability can be accounted for by a small number *k*, of the principal components. There is almost as much information in the *k* components as there is in the original dataset. The *k* principal components can then be used to replace the initial dataset.

Let the random vector $\mathbf{X = [X_1, X_2, \ldots, X_k]}$ have the covariance matrix l with eigenvalues $\lambda_1 > \lambda_2 > \cdots > \lambda_n > 0$. The principal components are given by:

$$P_1 = w_{01}X_0 = X_{w1} = w_{11}X_1 + w_{21}X_2 + \cdots + w_{n1}X_n$$

$$P_2 = w_{02}X_0 = X_{w2} = w_{12}X_1 + w_{22}X_2 + \cdots + w_{n2}X_n$$

.
.
.

$$P_m = w_{0m}X_0 = X_{wm} = w_{1m}X_1 + w_{2m}X_2 + \cdots + w_{nm}X_n$$

The *kth* principal component of the system is defined by:

$$P_k = w_{0k}X_0 = w_{1k}X_1 + w_{2k}X_2 + \cdots + w_{nk}X_n$$

Where X_i denotes the *ith* column of X, the historical input data on the *ith* variable in the system.

In matrix notation, we have the following:

$$P_m = Xw_m$$

2.0-2

Hence, the principal components are those linear combinations $P_1, P_2, \ldots, P_k$ whose variances are as large as possible. Each principal component is a time series of the transformed X variables, and the full $T \times n$ matrix of principal components, having P_m as its *mth* column may be written as

$$P = XW$$

2.0-3

However, W is an orthogonal matrix (since it is the matrix of eigenvectors), that is W ' =W−1 and so we have P'P = TΛ. P'P/T = Λ, gives the correlation matrix of the principal

components. This is a diagonal matrix, since Λ is the k × k diagonal matrix of eigenvalues. Where

$$\Lambda = \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \lambda_n \end{bmatrix}$$

2.0-4

In addition, the off-diagonal entries are all zero. This implies that the columns of P are uncorrelated, and the variance of the *mth* principal component is λm. Since the variance of each principal component is determined by its corresponding eigenvalue, the proportion of the total variation in X that is explained by the *mth* principal component is λ_m*/(sum of eigenvalues)*. Where the sum of the eigenvalues gives the total population variance, which is given by = $\lambda_1+\lambda_2+\cdots+\lambda_n$, and consequently the proportion of total variance due to the *mth* principal component is:

$$\frac{\lambda_m}{\lambda_1 + \lambda_2 + \cdots + \lambda_n}$$

2.0-5

Because of the choice of column labeling in W the principal components have been ordered so that P_1 belongs to the first and largest eigenvalue λ_1, P_2 belongs to the second largest eigenvalue λ_2, and so on. In a highly correlated system the first eigenvalue will be much larger than the others, as a result the first principal component will alone explain a large portion of the variability in the system. If most (for instance 80-90%) of the total population variance can be attributed to the first two or three principal components, then these components can essentially replace the original *k* variables without much loss of information.

3.0 ESTIMATING YIELD CURVE

3.1 Polynomials Approach

As we propose the following zero-coupon yield curve;

$$y^z(t) = \beta_0 + \sum_{i=1}^{n-1} \beta_i \frac{1}{(1+t)^i}$$

$$y^z(t) = \beta_0 + \sum_{i=1}^{n-1} \beta_i w^i$$

We can also write as

$$Y(t) = X(t)\mathrm{B}$$

Where

B is a column vector of *n* coefficients; $\mathrm{B} = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_{n-1} \end{bmatrix}$

X(*t*) is a row vector of *n-1* polynomials for time to maturity, *t;* $X(t) = \begin{bmatrix} w^0 & w^1 & \cdots & w^{n-1} \end{bmatrix}$ and w =1/(1+t)

The polynomial of 1/(1+t) is flexible and capable of rendering all of the common yield curve shapes. It is well behaves at both ends of the term structure,

$$y^z(0) = \sum_{i=0}^{n-1} \beta_i$$

$$y^z(\infty) = \beta_0$$

A parsimonious modeling approach require the yield curve to be specified with the smallest number of parameters that provide an acceptable to the observed bond data, models with three and four are good fit to the data. In estimation I generate eight orders of polynomials for each term to maturity for further analysis, but estimate term structure from polynomial of

1/(1+t) of four parameters, decided to estimate four parameter, it does not follow that one would use the first four polynomial terms.

3.2 Principal Component Analysis Approach

I proposed a principal component approach to selecting a *k* parameter model from possible *n* polynomial terms. We can write as

$$Y^P(t) = P(t)\alpha$$

Where,

a is a column vector of k coefficients $\alpha = \begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \vdots \\ \alpha_k \end{bmatrix}$,

P(t) is a row vector and $P(t) = X(t)A$

Let P and X be $T \times n$ matrices, composed of rows of P(t) and X(t) for t = 1 to T, respectively. So, we can write as

$$P = XW$$

Where, W is $n \times n$ matrix of eigenvectors of $C = X'X$ of the same size as W. Λ is $n \times n$ diagonal matrix of eigen values and each column of eigen values are in descending order with respect to eigen values and also compose the each column of eigenvector corresponding to eigen values. The principal component, *P*, contains *k* orthogonal vectors with the same number of observation as the original series that explain the maximum possible variance of the original series.

While polynomial of 1/(1+t) are naturally correlated, the principal components are not. A comparison of first three polynomials of 1/(1+t), and first three principal components of the polynomials is presented in Table 1 & 2 band Figure 1 & 2. It shows the high degree of correlation between the first three polynomials of 1/(1+t). This correlation is eliminated by combining these variables in to principal components.

Table 3 shows the estimated model and coefficients of principal component of the eight polynomial terms of 1/(1+t). I estimate three models of first three, two and one principal components. The intercept term of all models is constant. In Table 4 displays how the addition of each of the first three principal components provides considerable increase in the bond price residual sum of square residuals, because only first component is significant in all and it covers 93% of variation of original data.

In figure 3, the estimated zero coupon yield curve of 3-degree polynomial term of 1/(1+t) is increases from 2% to 3.87%, from term 6 months to 4 years and after 4 years, it is flat in shape. The term structure from first polynomial is much better approximation as compare to polynomial approach; it flexible and best fitted than the polynomial.

Principal Component's series are orthogonal to each other. In linear regression analysis orthgoanality is an advantage as variable are independent n that they can be added or subtracted from, a specification without altering the value of the, least square estimated parameters on the other include variable. This property is beneficial in one attempt to select an additional explanatory variable from number of candidate variable.

3.3 Conclusion

The simple polynomial form results in estimated models where the zero coupon yield diverge to plus or minus infinity as the term increases. The simple polynomial model is inconsistent with both theoretical consideration and observational reality.

I use small number of principal component of a larger number of polynomial of 1/(1+t) to produce an estimated zero coupon yield curve. Estimating a model via principal components enable us to take advantage of the principal component's lack of co linearity and promoted convergence in the non-linear estimation process.

The yield curve from polynomials of 1/(1+t) displayed tractable estimation properties of the yield curve modeled as polynomial of t. However, unlike the polynomial of *t*, the yield curve of polynomial of 1/(1+t) had stable long term yields.

As we see in the Figures 3, 4, 5 and 6 of term structure form four different methodologies, polynomials of 1/(1+t), principal components of polynomials of 1/(1+t) McCulloch cubic

spline and Nelson & Siegel methods. The term structure from simple polynomial of 1/(1+t) has no flexibility and converge very early at rate of approximately 4% at 4 term to maturity. The term structure from principal component (PCA) has much flexibility, especially at both ends, spot rate at 6 month is around 1.5% and it increase rapidly to 5 years maturity and then gradually. However, long rate from principal component approach is around 5.5%. Zero-coupon yield curve from methods, Nelson & Siegel and McCulloch cubic spline are flexible and spot rate for 6-months are 2% and 2.5% respectively.

	1/(1+t)	1/(1+t)^2	1/(1+t)^3
1/(1+t)	0.0249		
1/(1+t)^2	0.0164	0.0115	
1/(1+t)^3	0.0096	0.0070	0.0044

	1/(1+t)	1/(1+t)^2	1/(1+t)^3
1/(1+t)	1		
1/(1+t)^2	0.9710	1	
1/(1+t)^3	0.9165	0.9847	1

Table 1: Covariance and Correlation of polynomials of 1/(1+t)

	1st PC	2nd PC	3rd PC
1st PC	0.0422		
2nd PC	0.0000	0.0014	
3rd PC	0.0000	0.0000	0.0000

	1st PC	2nd PC	3rd PC
1st PC	1.0000		
2nd PC	0.0000	1.0000	
3rd PC	0.0000	0.0000	1.0000

Table 2: Covariance and Correlation of Principal Components

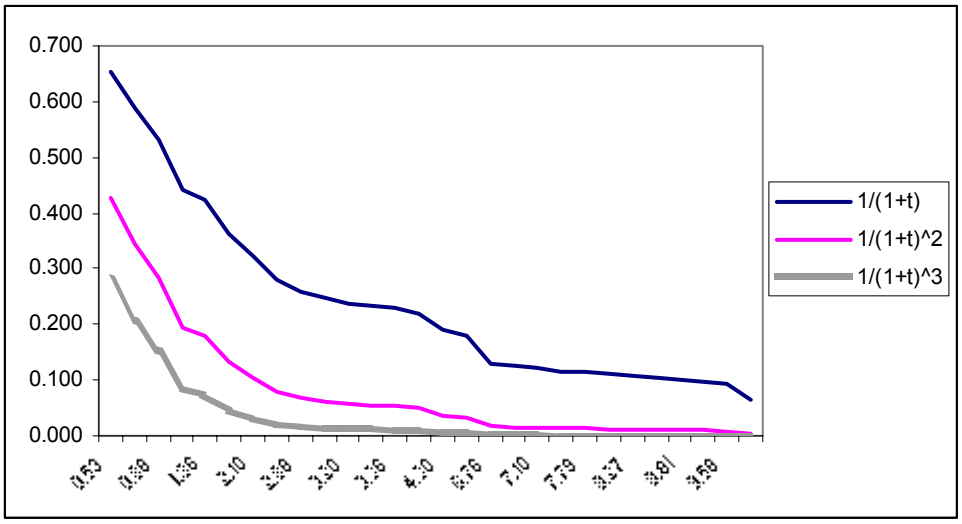

Figure 1: Plynomials of 1/(1+t)

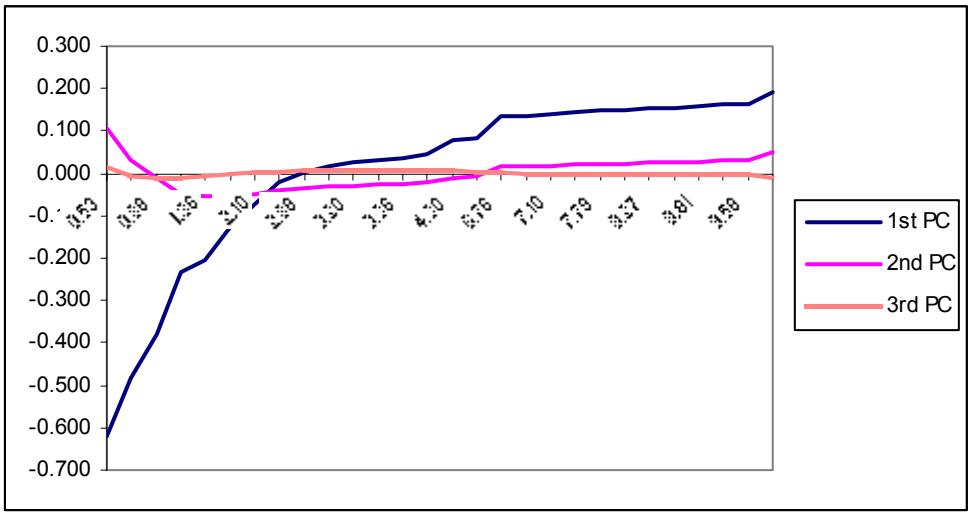

Figure 2: Principal Components of Polynomials of 1/(1+t)

	Ceofficients	Prob(β_i = 0)
β_0	0.04542	0.00000
β_1	0.00000	1.00000
β_2	0.00000	1.00000
β_3	0.88385	0.00214

	Ceofficients	Prob(β_i = 0)
β_0	0.04542	0.00000
β_1	0.00000	1.00000
β_2	-0.23022	0.00008

	Ceofficients	Prob(β_i = 0)
β_0	0.04542	0.00000
β_1	-0.07095	0.00002

Table 3: Estimated models parameter of yield curve from Principal Components

Sequential varibles	Bond price residual sum of squares
1st Principal Component	1780.639
2nd Principal Component	2093.71
3rd Principal Component	2720.834

Table 4: Zero curve estimation results from PCA

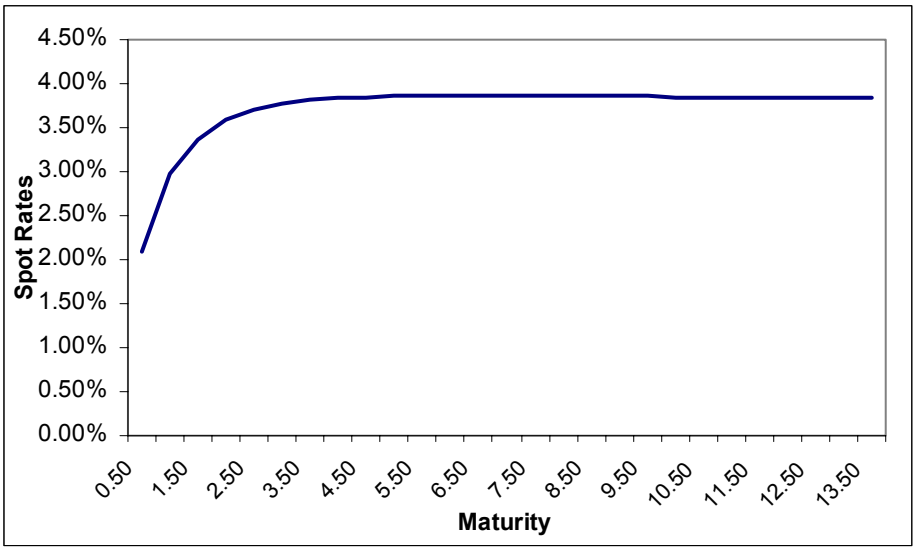

Figure 3: Estimated Zero yield curve from 3-degree polynomial

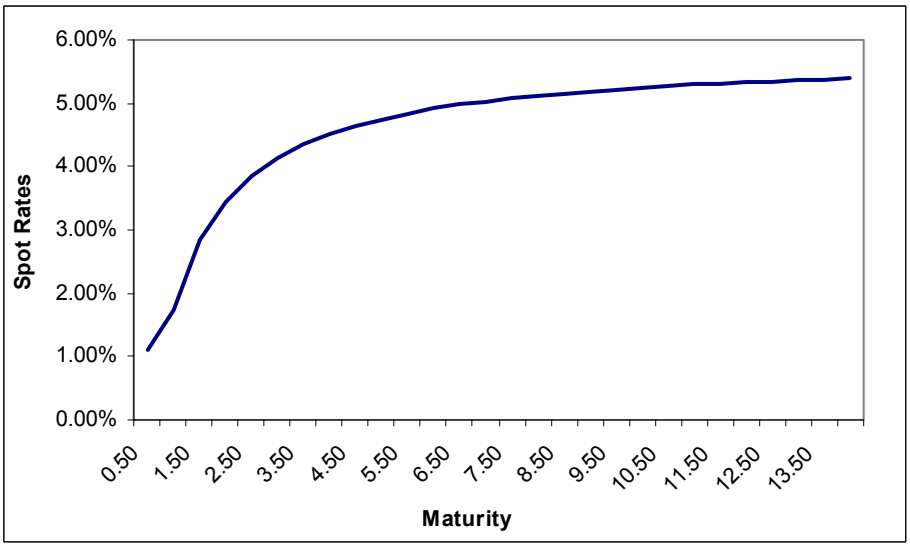

Figure 4: Estimated Zero curve from first Principal Components

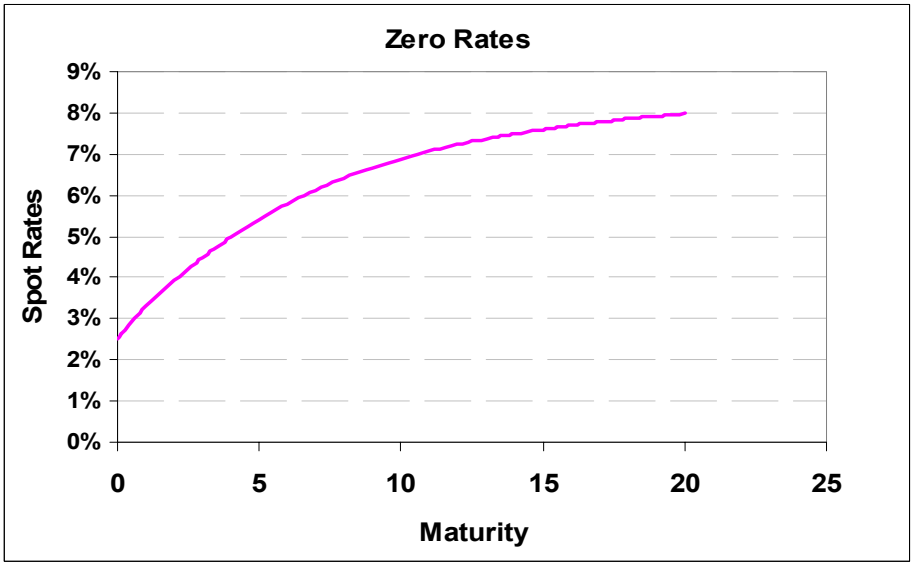

Figure 5: Estimated Zero curve from McCulloch Cubic Spline

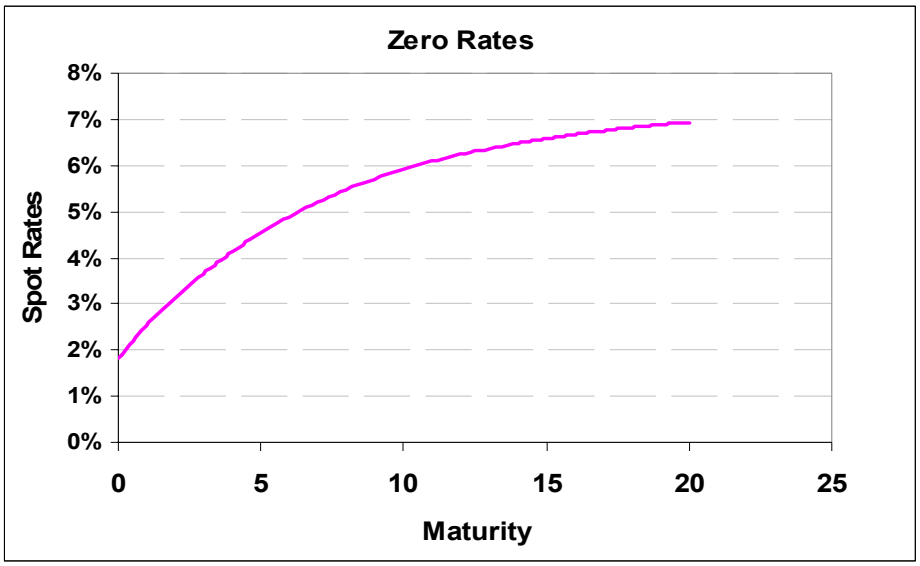

Figure 6: Estimated Zero curve from Nelson & Siegel

```
Sub Put_poly()
MLPutMatrix "Poly", Range("v3:ac30")
MatlabRequest
MLEvalString "[pc,zscores,pcvars] = princomp(Poly)"
MatlabRequest
MLGETMatrix "zscores", "Sheet1!AE3:Sheet1!AH30"
MatlabRequest
MLGETMatrix "pcvars", "AO3:AO10"


End Sub
```

```
Sub matlab2()
MLPutMatrix "W", Range("w")
MatlabRequest
MLPutMatrix "TERM", Range("term")
MatlabRequest
MLPutMatrix "data", Range("D3:D30")
MatlabRequest
MLPutMatrix "x0", Range("S19:S22")
MatlabRequest
MLEvalString "yydata = data"
MatlabRequest
MLEvalString "[x,resnorm]= lsqcurvefit(@myfun,x0,TERM,yydata)"
MLGETMatrix "x", "S19:S22"
MatlabRequest
MLGETMatrix "resnorm", "S25"
MatlabRequest
End Sub
```

```
Sub SolverMacro()

' Example Solver VBA Macro

    SolverOk SetCell:="$s$10", _
             MaxMinVal:=2, _
             ValueOf:="0", _
             ByChange:="$s$7:$s$10"
    SolverSolve userFinish:=True
End Sub
```

Figure 7: Coding of Excel macros by using Matlab commands

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